LPTHE 01-25
hep-th/0105210
May, 2001

Superbranes and Super Born-Infeld Theories as Nonlinear Realizations

E. Ivanov [a,b]

[a] *Bogoliubov Laboratory of Theoretical Physics, JINR,*
141 980 Dubna, Moscow Region, Russia

[b] *Laboratoire de Physique Théorique et des Hautes Energies,*
Unité associée au CNRS UMR 7589, Université Paris 7
2 Place Jussieu, 75251 Paris Cedex 05, France

Abstract

We outline, on a few instructive examples, the characteristic features of the approach to superbranes and super Born-Infeld theories based on the concept of partial spontaneous breaking of global supersymmetry (PBGS). The examples include the $N = 1, D = 4$ supermembrane and the "space-filling" D2- and D3-branes. Besides giving a short account of the available results for these systems, we present some new developments. For the supermembrane we prove the equivalence of the equation of motion following from the off-shell Goldstone superfield action and the one derived directly from the nonlinear realizations formalism. We give a new derivation of the off-shell Goldstone superfield actions for the considered systems, using a universal procedure inspired by the relationship between linear and nonlinear realizations of PBGS.

Submitted to Proceedings of the Seminar "Classical and Quantum Integrable Systems" dedicated to the memory of M.V. Saveliev (Protvino, Russia, 8–11 January 2001)

1 Introduction

One of the approaches to superbranes proceeds from the concept of partial breaking of global supersymmetry (PBGS) [1], [2]-[19]. In such a description the objects representing the physical worldvolume superbrane degrees of freedom are Goldstone superfields. The worldvolume supersymmetry acts on them as linear transformations and so is manifest. The rest of the full target supersymmetry is realized nonlinearly. After passing to components in the Goldstone superfield action and eliminating auxiliary fields, one recovers a "static-gauge" form of the appropriate Green-Schwarz-type action (in general, after a field redefinition).

While for the ordinary p-branes the worldvolume multiplets are scalar, analogous supermultiplets of Dp-branes are known to be vector, with the Born-Infeld dynamics for gauge fields (see [20] and refs. therein). So the corresponding PBGS actions should form a subclass of manifestly supersymmetric extensions [21]-[23] of the Born-Infeld (BI) action. The actions from this variety are characterized by the second nonlinearly realized hidden supersymmetry. The PBGS approach can be considered as an efficient tool for deducing such superextensions of the BI action. Until now, only superextensions of abelian BI theory were derived in this way [5, 7, 12, 13]. However, this approach could be useful in the non-abelian case too.[1]

Here we explain, on a few instructive examples ($N = 1$ supermembrane, space-filling D2- and D3-branes), how the PBGS approach augmented with the general methods of the theory of nonlinear realizations [25] leads to a manifestly supersymmetric description of superbranes and superextensions of the BI theory in terms of worldvolume superfields. What can be directly deduced from the nonlinear realizations formalism in most cases, is the Goldstone superfield dynamical equations describing the given superbrane or super BI theory on shell. The construction of the off-shell superfield actions is more tricky, and it requires constructing a *linear* realization of the corresponding PBGS pattern. The superbrane or BI superfield Lagrangian density is identified with a proper component of some linear supermultiplet of the full underlying supersymmetry. This multiplet is subjected to covariant constraints which express all its components in terms of the Goldstone multiplet of the unbroken supersymmetry. The precise form of these constraints can be found using the general relationship between linear and nonlinear realizations of supersymmetries [26] adapted to the PBGS case in [17, 18].[2] We apply this universal procedure to give a new derivation of the off-shell Goldstone superfield actions for the examples considered. Also, in the supermembrane case we prove the equivalence of the superfield equations of motion derived from the off-shell action and those postulated in the pure nonlinear realizations framework [7, 11].

2 N=1, D=4 supermembrane

2.1 $N = 1, D = 4$ supermembrane as a PBGS system. The supermembrane in $D = 4$ spontaneously breaks half of the $N = 1, D = 4$ supersymmetry and one translation. The set of generators of $N = 1$ $D = 4$ Poincaré superalgebra in the $d = 3$ notation is naturally split into the unbroken $\{Q_a, P_{ab}\}$ and broken $\{S_a, Z\}$ parts ($a, b = 1, 2$). The basic anti-commutation

[1] Its "inborn" feature (as distinct, e.g., from the approach proceeding from gauge-fixed Green-Schwarz Dp-brane actions [24]) is the manifestly *linear* realization of unbroken supersymmetry.

[2] A different, though seemingly equivalent adaptation was used in a recent preprint [19].

relations in this notation read

$$\{Q_a, Q_b\} = \{S_a, S_b\} = P_{ab},\ \{Q_a, S_b\} = \epsilon_{ab} Z\ . \tag{2.1}$$

As was argued in [11], for deriving manifestly covariant superfield equations describing the worldvolume dynamics of superbrane in the present case (and some other ones), it suffices to deal with a nonlinear realization of the superalgebra (2.1) itself, ignoring all generators of the automorphisms of (2.1). So we put all generators into the coset and associate the $N=1\,,\,d=3$ superspace coordinates $\left\{\theta^a, x^{ab}\right\}$ with Q_a, P_{ab}. The remaining coset parameters are Goldstone superfields, $\psi^a \equiv \psi^a(x,\theta),\ q \equiv q(x,\theta)$. A coset element g is defined by [3]

$$g = e^{x^{ab}P_{ab}} e^{\theta^a Q_a} e^{qZ} e^{\psi^a S_a}\ . \tag{2.2}$$

Then one constructs the Cartan 1-forms

$$g^{-1}dg = \omega_Q^a Q_a + \omega_P^{ab} P_{ab} + \omega_Z Z + \omega_S^a S_a, \tag{2.3}$$

$$\omega_P^{ab} = dx^{ab} + \frac{1}{4}\theta^{(a} d\theta^{b)} + \frac{1}{4}\psi^{(a} d\psi^{b)}\ ,\ \ \omega_Z = dq + \psi_a d\theta^a,\ \ \omega_Q^a = d\theta^a,\ \ \omega_S^a = d\psi^a \tag{2.4}$$

and the corresponding covariant derivatives

$$\mathcal{D}_{ab} = (E^{-1})_{ab}^{cd}\,\partial_{cd}, \quad \mathcal{D}_a = D_a + \frac{1}{2}\psi^b D_a \psi^c\, \mathcal{D}_{bc}, \tag{2.5}$$

where

$$E_{ab}^{cd} = \frac{1}{2}(\delta_a^c \delta_b^d + \delta_a^d \delta_b^c) + \frac{1}{4}(\psi^c \partial_{ab}\psi^d + \psi^d \partial_{ab}\psi^c)\ ,\ \ D_a = \frac{\partial}{\partial\theta^a} + \frac{1}{2}\theta^b \partial_{ab},\ \ \{D_a, D_b\} = \partial_{ab}\ . \tag{2.6}$$

The set of Goldstone superfields $\{q(x,\theta), \psi^a(x,\theta)\}$ is reducible. Indeed, ψ_a appears inside the form ω_Z *linearly* and so it can be covariantly eliminated by imposing the following manifestly covariant inverse Higgs [27] constraint

$$\omega_Z|_{d\theta} = 0 \Rightarrow \psi_a = \mathcal{D}_a q\ , \tag{2.7}$$

where $|_{d\theta}$ means the ordinary $d\theta$-projection of the form. Thus $q(x,\theta)$ is the only essential Goldstone superfield needed to describe the partial spontaneous breaking $N=1\,,\ D=4\ \Rightarrow N=1\,,\ d=3$ within the coset scheme.

In order to get dynamical equations, we put an additional, manifestly covariant constraint on the superfield $q(x,\theta)$. It is a direct covariantization of the "flat" equation of motion:

$$D^a D_a q = 0 \quad \Rightarrow \quad \mathcal{D}^a \mathcal{D}_a q = 0\ . \tag{2.8}$$

Eq. (2.8) coincides with the dynamical equation of the supermembrane in $D=4$ as it was given in [7]. It was derived there from the coset approach with the $D=4$ Lorentz group generators included, so (2.8) actually possesses the hidden covariance under the full $D=4$ Lorentz group $SO(1,3)$. For the bosonic field $q(x) \equiv q(x,\theta)|_{\theta=0}$ it yields the equation corresponding to the static-gauge form of the Nambu-Goto action for membrane in $D=4$. Below we shall prove that (2.8) is equivalent to the equation following from an off-shell action of the supermembrane.

[3] In our notation the coset parameters x^{ab} and q are imaginary, while θ^a and ψ^a are real.

Our next goal will be to give a new construction of this invariant off-shell superfield action. As distinct from ref. [7], here we apply the systematic approach based on the relationship between linear and nonlinear realizations of supersymmetry [26]. The construction is quite similar to the one exploited in [18] in application to $d = 2$ PBGS systems.

As a first step, we define a *linear* realization of the considered PBGS pattern $N = 1, D = 4 \rightarrow N = 1, d = 3$. From the $d = 3$ point of view, it amounts to $N = 2 \rightarrow N = 1$, with the $N = 2, d = 3$ Poincaré superalgebra given by the relations (2.1). The primary object of such a realization is the scalar chiral $N = 2, d = 3$ superfield $\Phi(x, \theta, \zeta)$, where $x^{ab}, \theta^a, \zeta^d$ are the $N = 2, d = 3$ superspace coordinates. It is assumed to have the following transformation property under the central charge operator Z:

$$Z\Phi = 1 \ . \tag{2.9}$$

This means that the central charge generator acts as shifts of Φ. Such a realization can be understood as the following specific coset realization of $N = 2, d = 3$ supersymmetry (2.1): one treats Φ as the coset parameter (Goldstone superfield) associated with Z, while the rest of coset parameters as the coordinates of $N = 2, d = 3$ superspace on which Φ "lives" (cf. similar $d = 2$ realizations considered in [18]). With respect to the $N = 1$ supersymmetry $\{P_{ab}, Q_a\}$, the superfield Φ is a collection of standard $N = 1$ superfields (the coefficients in the expansion of Φ in ζ^a), while under the S-supersymmetry it transforms in the following way

$$\delta_\eta \Phi = -\eta^a \left(\frac{\partial}{\partial \zeta^a} - \frac{1}{2}\zeta^b \partial_{ab} - \theta_a Z \right) \Phi \ . \tag{2.10}$$

Respectively, the spinor covariant derivatives in this realization are given by

$$\hat{D}^\theta_a = \frac{\partial}{\partial \theta^a} + \frac{1}{2}\theta^b \partial_{ab} - \zeta_a Z = D_a - \zeta_a Z \ , \quad D^\zeta_a = \frac{\partial}{\partial \zeta^a} + \frac{1}{2}\zeta^b \partial_{ab} \ . \tag{2.11}$$

The covariant chirality condition reads

$$\left(\hat{D}^\theta_a - i D^\zeta_a \right) \Phi = 0 \quad \Rightarrow \quad \Phi = \phi - i\zeta^a D_a \phi + \frac{1}{4}\zeta^2 \left[D^2\, \phi + 2i \right] \ , \ \ \phi \equiv \phi(x, \theta) \ , \tag{2.12}$$

where (2.9) was taken into account. Thus the complex $N = 1$ superfield $\phi(x, \theta)$ accommodates the irreducible set of the $(4 + 4)$ off-shell component fields of $\Phi(x, \theta, \zeta)$. Its S-supersymmetry transformation directly stems from (2.10) and (2.9):

$$\delta_\eta \phi = \eta^a \theta_a + i\eta^a D_a \phi \ . \tag{2.13}$$

For the real superfields ρ and ϕ_0 defined by

$$\phi = \phi_0 + i\rho$$

we obtain the following transformation laws

$$\delta_\eta \rho = -i\eta^a \theta_a + \eta^a D_a \phi_0 \ , \quad \delta_\eta \phi_0 = -\eta^a D_a \rho \ . \tag{2.14}$$

The spinor superfield

$$\xi_a = i D_a \rho$$

transforms under the S-supersymmetry with an inhomogeneous shift

$$\delta_\eta \xi_a = \eta_a \left(1 - \frac{i}{2} D^2 \phi_0\right) - \frac{i}{2} \eta^b \partial_{ab} \phi_0 \ , \tag{2.15}$$

and so can be viewed as the Goldstone fermion of linear realization of the same PBGS pattern $N = 2 \to N = 1, d = 3$. The field content of $\rho(x, \theta)$ coincides with that of $q(x, \theta)$, so ρ can be regarded as the $N = 1$ Goldstone superfield for the spontaneously broken Z-transformations (it is shifted under Z). It is interesting to note that the role of the inverse Higgs constraints in the linear realization is played by the chirality condition (2.12) which expresses all $N = 1$ superfield components of $\Phi(x, \theta, \zeta)$ via derivatives of ρ and ϕ_0.

Besides the basic Goldstone superfield ρ, there still remains the superfield ϕ_0 possessing homogeneous transformation laws under both $N = 1, d = 3$ supersymmetries. Now we shall show that it can be eliminated in terms of ρ by imposing a nonlinear constraint which brings the considered linear realization into a nonlinear one related to the original nonlinear realization by a field redefinition. To this end, we apply the method of refs. [26], [17, 18] to the system of $N = 1$ superfields ξ_a, ϕ_0. Construct their *finite* S-supersymmetry transformation and replace, in the final expressions, the parameters η^a by the Goldstone superfields $\psi^a(x, \theta)$ of the original nonlinear realization (taken with the sign minus). The resulting objects

$$\begin{aligned} \tilde{\xi}_a &= \xi_a - \psi_a \left(1 - \frac{i}{2} D^2 \phi_0\right) + \frac{i}{2} \psi^d \partial_{ad} \phi_0 - \frac{1}{4} \psi^2 \partial_{ab} \xi^b \ , \\ \tilde{\phi}_0 &= \phi_0 - i \psi^a \xi_a + \frac{i}{2} \psi^2 \left(1 - \frac{i}{2} D^2 \phi_0\right) \end{aligned} \tag{2.16}$$

are homogeneously transformed under the S-supersymmetry (and under the Q one, of course). So it is the covariant condition to put them equal to zero

$$\tilde{\xi}_a = 0 \ , \quad \tilde{\phi}_0 = 0 \ . \tag{2.17}$$

Using the nilpotency property $\psi^3 = 0$, it is easy to find that these equations amount to

$$\text{(a)} \ \ \psi^a = \frac{\xi^a}{1 - \frac{i}{2} D^2 \phi_0} \ , \quad \text{(b)} \ \ \phi_0 = \frac{i}{2} \psi^2 \left(1 - \frac{i}{2} D^2 \phi_0\right) = \frac{i}{2} \frac{\xi^2}{1 - \frac{i}{2} D^2 \phi_0} \ . \tag{2.18}$$

These relations coincide (up to a slight difference in the notation) with those found in [7]. The first one is the equivalence relation between the nonlinear and linear realizations Goldstone fermions, while the second one serves to express ϕ_0 in terms of ψ^a or ξ^a:

$$\phi_0 = \frac{i}{2} \frac{\psi^2}{1 - \frac{1}{4} D^2 \psi^2} = \frac{i\, \xi^2}{1 + \sqrt{1 + D^2 \xi^2}} \ . \tag{2.19}$$

In view of the transformation property (2.14) of ϕ_0, the integral

$$S \sim \int d^3x d^2\theta \, \phi_0 \tag{2.20}$$

is invariant with respect to the whole $N = 2, d = 3$ supersymmetry, and so it is the sought off-shell action of the Goldstone superfield $\rho(x, \theta)$ (or $q(x, \theta)$). It describes, in a manifestly worldvolume supersymmetric manner, the $N = 1, D = 4$ supermembrane in a flat background.

It can equally be written through the initial chiral $N=2$ superfield $\Phi(x,\theta,\zeta)$, eq. (2.12), as an integral over the full or chiral $N=2, d=3$ superspaces

$$S \sim \int d^3x d^2\theta d^2\zeta \Phi\bar{\Phi} \sim \int d^3x_L d^2\theta_L \Phi + \text{c.c.} \ . \tag{2.21}$$

Here $\theta^a_L = \theta^a - i\zeta^a$, $x^{ab}_L = x^{ab} + \frac{i}{2}\theta^{(a}\zeta^{b)}$, one should substitute into (2.12) the explicit expression for ϕ_0 and, in the second case, pass to the chiral basis in which Φ does not depend on $\theta^a_R = \theta^a + i\zeta^a$. In such a representation the full $N=2$, $d=3$ supersymmetry (2.1) is manifest. Note that the two invariants (2.21) are independent before passing to the nonlinear realization, but they become identical (up to a numerical factor) in the nonlinear realization framework. A similar phenomenon takes place in other PBGS cases [6, 20, 18].

Note that the same relations (2.18) could be obtained by imposing, in the spirit of [6], the nilpotency condition on the appropriate real $N=2$ superfield constructed out of Φ and its conjugate and having a zero central charge [16, 19]. Our purpose here was to demonstrate that the generic method used earlier in refs [17, 18] works fairly well in this case too.

2.2 Equivalence of two forms of the supermembrane equations of motion. Here we show that the dynamical equation (2.8) postulated on the purely geometric grounds and the equation of motion following from the off-shell action (2.20) are equivalent to each other.

Eq. (2.8) written in terms of the Goldstone fermion superfield ψ_a

$$\mathcal{D}^a\psi_a = 0 \ , \tag{2.22}$$

can be cast in the following more detailed form

$$W + \frac{1}{2}\psi^a W^{bc}\partial_{ab}\psi_c - \frac{1}{16}\psi^2 W^{ac}\partial_{bc}\psi_d\partial^{bd}\psi_a = 0 \ , \tag{2.23}$$

where

$$W \equiv D^a\psi_a \ , \quad W^{ab} \equiv D^{(a}\psi^{b)} \tag{2.24}$$

and the explicit expression (2.5) for the covariant derivative $\mathcal{D}_a$ was used. In such a form the equation includes only flat derivatives D_a and ∂_{ab}.

On the other hand, the equation of motion which follows from the off-shell action (2.20) by varying it with respect to the unconstrained $N=1$ superfield ρ can be written in terms of ψ^a

$$D^a\lambda_a = 0 \ , \tag{2.25}$$

where

$$\lambda_a \equiv \frac{\psi_a}{1-\frac{1}{2}D\psi\cdot D\psi} + \frac{1}{4}\frac{\psi^2}{(1-\frac{1}{2}D\psi\cdot D\psi)^2}\left\{\partial_{ab}\psi^b + 2D_b\psi_a D_d\psi_c\partial^{db}\psi^c\right\} \tag{2.26}$$

and $A\cdot B \equiv A^{ab}B_{ab}$. This form of the equation can be deduced from the equivalence relation (2.18). Let us start from the Lagrangian (2.19) in the form

$$\phi_0 = \frac{i}{2}\frac{\xi^2}{1-\frac{i}{2}D^2\phi_0} = \frac{i}{2}\psi^2(1-\frac{i}{2}D^2\phi_0) \ , \quad S \sim \int d^3x d^2\theta \ \phi_0(x,\theta) \tag{2.27}$$

with

$$(1-\frac{i}{2}D^2\phi_0)_{eff} = \frac{1}{1+\frac{1}{2}D\psi\cdot D\psi} \ . \tag{2.28}$$

Here the subscript "*eff*" means that we ignore all terms $\sim \psi^a$ (but not those with derivatives on ψ^a). The variation of ϕ_0 reads

$$\delta\phi_0 = i\delta\xi^a\psi_a - \frac{1}{4}\psi^2(D^2\delta\phi_0)_{eff} \ , \tag{2.29}$$

where $(D^2\delta\phi_0)_{eff}$ is determined from the equation

$$(D^2\delta\phi_0)_{eff}\left(1 - \frac{1}{2}D\psi\cdot D\psi\right) = -2i\, D^l\delta\xi^a D_l\psi_a + i\delta\xi^a D^2\psi_a \ . \tag{2.30}$$

Using eq. (2.18) and the kinematical constraint

$$D^2\xi_a = \partial_{ab}\xi^b \tag{2.31}$$

following from the relation $\xi_a = iD_a\rho$, one obtains, in zeroth order in ψ,

$$D^2\psi_a = \partial_{ab}\psi^b + D_b\psi_a D_d\psi_c\partial^{bd}\psi^c + O(\psi) \ . \tag{2.32}$$

After substituting all this in (2.29) and integrating in $\delta_\eta S$ by parts with keeping in mind that $\delta\xi^a = iD^a\delta\rho$, one obtains the equation of motion in the form (2.25), (2.26).

Let us prove the equivalence of eqs. (2.23) and (2.25). First, we shall show that (2.25) is identically satisfied if (2.23) holds.

As a preparatory step, we extract some corollaries of (2.23). Acting on (2.23) by spinor derivative, one gets, in the zeroth order in ψ

$$D^2\psi_a = -\partial_{ab}\psi^b - W_{ab}W_{cd}\partial^{bd}\psi^c + O(\psi) \ . \tag{2.33}$$

Comparing it with the kinematical constraint (2.32), one finds

$$\text{(a)}\ \ D^2\psi_a = O(\psi) \ , \quad \text{(b)}\ \ \partial_{ab}\psi^b + W_{ab}W_{cd}\partial^{bd}\psi^c = O(\psi) \ . \tag{2.34}$$

Further, in the same order, acting on (2.23) by D^2 ($D^2D^a = -\partial^{ab}D_b$), one obtains

$$\partial^{ab}W_{ab} = -W^{ab}W^{dc}\partial_{bc}W_{ad} + O(\psi) \ , \tag{2.35}$$

where eq.(2.34b) was used.

Let us apply to eq. (2.25). Firstly, owing to eq.(2.34b) one can make in (2.25) the substitution

$$\lambda_a \ \Rightarrow \ \frac{\psi_a}{1 - \frac{1}{2}D\psi\cdot D\psi} - \frac{1}{4}\frac{\psi^2}{(1 - \frac{1}{2}D\psi\cdot D\psi)^2}\,\partial_{ab}\psi^b \ . \tag{2.36}$$

Secondly, because of the relation $W^2 \sim \psi^2$ following from (2.23), one can replace altogether

$$(D\psi\cdot D\psi) \ \Rightarrow \ (W\cdot W) \ . \tag{2.37}$$

After expressing W from (2.23) and using eq. (2.34a) together with the identity

$$\partial_{ab}\psi^b\partial^{ad}\psi^c W_{dc} = O(\psi) \tag{2.38}$$

which follows from (2.34b), eq. (2.25) can be rewritten, up to the non-singular factor

$$1 - \frac{1}{2}(W\cdot W) \equiv F,$$

in the following equivalent form

$$(W\cdot W)\,\psi^a\partial_{ab}\psi_c\,W^{bc}+\psi^2\,W^{ab}W^{dc}\partial_{bc}W_{ad}-2(G+K)=0\ , \tag{2.39}$$
$$G\equiv\psi_a\,W^{ab}D^2\psi_b\ ,\quad K\equiv\psi_a\,W^{ab}\,\partial_{bc}\psi^c\ . \tag{2.40}$$

It remains to compute G and K. For this one should know $D^2\psi^a$ and $\partial_{ab}\psi^b$ up to the terms $\sim\psi$, while eqs. (2.34) define them only up to the terms ~ 1. To find these quantities, one should evaluate $D^2\psi^a$ at the required order both from the kinematical constraint (using eqs. (2.18) and (2.31)) and from (2.23) and compare these two expressions. They can be straightforwardly computed, but look rather cumbersome. Much simpler are the corresponding expressions for G and K. Starting from the kinematical eqs. (2.31), (2.18), expressing W from (2.23) and using the dynamical identity (2.38) combined with cyclic identities, one finds

$$G=K-\frac{1}{2}(W\cdot W)\,\psi^a\,W^{bc}\,\partial_{ab}\psi_c+\frac{1}{2}\psi^2\,W^{ab}\,W^{cd}\,\partial_{ac}W_{bd}\ . \tag{2.41}$$

On the other hand, the direct calculation making use only of eq. (2.23) and its corollaries yields

$$G=-K+\frac{1}{2}(W\cdot W)\,\psi^a\,W^{bc}\,\partial_{ab}\psi_c+\frac{1}{2}\psi^2\,W^{ab}\,W^{cd}\,\partial_{ac}W_{bd}\ . \tag{2.42}$$

Comparing these expressions, one finds

$$G=\frac{1}{2}\psi^2\,W^{ab}\,W^{cd}\,\partial_{ac}W_{bd}\ ,\quad K=\frac{1}{2}(W\cdot W)\,\psi^a\,W^{bc}\,\partial_{ab}\psi_c\ . \tag{2.43}$$

Substituting them into eq. (2.39), one finds the latter to be identically satisfied. Thus we have shown that eq. (2.25) is fulfilled as a consequence of eq. (2.23).

To prove the equivalence of eqs. (2.23), (2.25), it remains to show that eq. (2.25) necessarily implies (2.23). As a first step, one observes that (2.25) implies the same relations (2.33), (2.35), (2.34), (2.38) as eq. (2.23) and so we are allowed to use them in the course of the proof. In particular, using in addition the fact that $W\sim\psi,\psi^2$ as another corollary of (2.25), one can make the replacements (2.36), (2.37) in (2.25). Further, from the kinematical constraints and (2.25) one derives the *same* expressions (2.43) for G and K (obtained earlier starting from (2.23)). Substituting them into (2.25), one gets

$$W+\frac{1}{2}\psi^aW^{bc}\partial_{ab}\psi_c=0 \tag{2.44}$$

that actually amounts to eq. (2.23) due to the relation (2.34b) and its corollary (2.38).

Thus we have proven the equivalence of eq. (2.23) derived from the purely geometric setting and eq. (2.25) obtained from the variation principle associated with the off-shell action (2.27). An important consequence of this fact is that the action (2.27) and eq. (2.25) possess all symmetries encoded in (2.23), including the $D=4$ Lorentz symmetry.

3 Space-filling D2-brane

As the second instructive example, we consider the "space-filling" D2-brane with the $N=1,\ d=3$ vector multiplet as the worldvolume one.

3.1 D2-brane dynamics from nonlinear realizations. Our starting point is the superalgebra (2.1) with $Z = 0$. The coset element g contains only one Goldstone superfield ψ^a, and the covariant derivatives are still given by (2.5). In the flat case the $d = 3$ vector multiplet is described by a $N = 1$ spinor superfield strength μ_a subjected to the Bianchi identity:

$$D^a \mu_a = 0 \ \Rightarrow \ \left\{ \begin{array}{l} D^2 \mu_a = -\partial_{ab}\mu^b \ , \\ \partial_{ab} D^a \mu^b = 0 \ . \end{array} \right. \tag{3.1}$$

Its equation of motion reads

$$D^2 \mu_a = 0 \ . \tag{3.2}$$

It was shown in [11] that the following manifestly covariant generalization of (3.1), (3.2) describes the D2-brane:

$$\text{(a)} \ \ \mathcal{D}^a \psi_a = 0 \ , \quad \text{(b)} \ \ \mathcal{D}^2 \psi_a = 0 \ . \tag{3.3}$$

The reasoning was mainly based on the observation that the purely bosonic limit of (3.3) amounts to the following equation for the vector $V_{ab} \equiv \mathcal{D}_a \psi_b|_{\theta=0}$:

$$\left(\partial_{ac} + V_a^d V_c^f \partial_{df} \right) V_b^c = 0 \ . \tag{3.4}$$

This nonlinear but polynomial equation was shown to be a “disguised” form of the equations of the non-polynomial $d = 3$ BI action which is just the bosonic core of the superfield D2-brane PBGS action as was explicitly demonstrated in [7]. The passing to the standard form of the $d = 3$ BI equation is achieved by a field redefinition which is a bosonic limit of the superfield equivalence redefinition relating the nonlinear realization Goldstone fermion ψ_a to μ_a treated as the Goldstone fermion of a *linear* realization of the same PBGS pattern (see next Subsection). Using this equivalence, one may explicitly show, like in the supermembrane case, that the equations (3.3) are equivalent to the worldvolume superfield equation following from the off-shell D2-brane action given in [7].

3.2 Off-shell superfield D2-brane action. Now we shall re-derive the off-shell D2-action of ref. [7] by the same generic method which was applied above to construct the Goldstone superfield action of $N = 1, D = 4$ supermembrane. To define the appropriate linear realization of the considered PBGS pattern, one needs to embed the $N = 1, d = 3$ Maxwell superfield strength μ_a into a linear $N = 2, d = 3$ multiplet. The latter should have such a transformation law under the S-supersymmetry that μ_a transform with an inhomogeneous term $\sim \eta_a$ and so admit an interpretation as the Goldstone fermion of linear realization.

The appropriate $N = 2, d = 3$ supermultiplet was proposed in [16] as a deformation of the $N = 2, d = 3$ Maxwell multiplet (which is a dimensional reduction of the $N = 1, d = 4$ tensor multiplet). In our notation this deformed multiplet is described by a real $N = 2, d = 3$ superfield $W(x, \theta, \zeta)$ subjected to the following constraints

$$\text{(a)} \ \ \left[(D)^2 - (D^\zeta)^2 \right] W = -2i \ , \quad \text{(b)} \ \ D^a D^\zeta_a W = 0 \tag{3.5}$$

(this form of constraints is most convenient for our purposes, it can be obtained from the one given in [16] by choosing a specific frame with respect to the explicitly broken $U(1)$-automorphism symmetry and making an appropriate rescaling of W [4]).

[4]For the first time such a deformation of the $N = 1, d = 4$ tensor multiplet constraints was considered in [28] in the context of $N = 4$ superconformal mechanics.

The standard S-supersymmetry transformation law of W

$$\delta_\eta W = -\eta^a \left(\frac{\partial}{\partial \zeta^a} - \frac{1}{2}\zeta^b \partial_{ab} \right) W \tag{3.6}$$

implies the following transformation laws for the irreducible $N = 1$ superfield components of $W(x, \theta, \zeta)$, $\mu_a \equiv -iD^\zeta_a W|_{\zeta=0}$ and $w \equiv W|_{\zeta=0}$,

$$\text{(a)}\ \ \delta_\eta \mu_a = \eta_a \left(1 - \frac{i}{2}D^2 w\right) + \frac{i}{2}\eta^b \partial_{ab} w \ , \quad \text{(b)}\ \ \delta_\eta w = -i\eta^a \mu_a \ . \tag{3.7}$$

It is easy to check that eq. (3.7a) is consistent with the Bianchi identity (3.1) (which is none other than eq. (3.5b)). Just due to the presence of constant $U(1)_A$ breaking term in the r.h.s. of (3.5a), the $N = 1$ Maxwell superfield μ_a transforms inhomogeneously under the S-supersymmetry, and thus is recognized as the Goldstone fermion of the linear realization of the considered $N = 2 \to N = 1, d = 3$ PBGS pattern.

Like in the supermembrane case, the additional homogeneously transforming $N = 1$ superfield $w(x, \theta)$ can be traded for the Goldstone-Maxwell one μ_a by imposing nonlinear constraints the precise form of which is dictated by our generic method applied to the given system. As the first step, one defines the superfields $\tilde{\mu}_a$ and $\tilde{w}$ as finite S-supersymmetry transforms of μ_a and w, with the supertranslation parameter η^a being replaced by $-\psi^a(x, \theta)$

$$\tilde{\mu}_a = \mu_a - \psi_a \left(1 - \frac{i}{2}D^2 w\right) - \frac{i}{2}\psi^b \partial_{ab} w - \frac{1}{4}\psi^2 \partial_{ab}\mu^b \ , \ \tilde{w} = w + i\psi^a \mu_a - \frac{i}{2}\psi^2 \left(1 - \frac{i}{2}D^2 w\right) . \tag{3.8}$$

These quantities homogeneously transform under all $N = 2, d = 3$ transformations and so one can covariantly equate them to zero

$$\tilde{\mu}_a = \tilde{w} = 0 \ . \tag{3.9}$$

From these covariant constraints one gets the equivalence relation between ψ^a and μ^a

$$\psi^a = \frac{\mu^a}{1 - \frac{i}{2}D^2 w} \ , \tag{3.10}$$

as well as the relation

$$w = -\frac{i}{2}\frac{\mu^2}{1 - \frac{i}{2}D^2 w} \ . \tag{3.11}$$

These are precisely the equations derived in [7] (up to a rescaling of w). They can be used to express w in terms of either ψ^a, or μ^a

$$w = -\frac{i}{2}\frac{\psi^2}{1 + \frac{1}{4}D^2\psi^2} = -\frac{i\,\mu^2}{1 + \sqrt{1 - D^2\mu^2}} \ . \tag{3.12}$$

This composite superfield is just the corresponding Goldstone superfield Lagrangian density,

$$S \sim \int d^3x d^2\theta \, w \ , \tag{3.13}$$

since, in virtue of the Bianchi identity (3.1), the $d^3xd^2\theta$ integral of the variation (3.7b) is vanishing, i.e. $\delta_\eta S = 0$.

The same superfield D2-brane action can be written in a manifestly $N = 2$ supersymmetric form as an integral over the whole $N = 2$ superspace, with either W^2 or the $N = 2, d = 3$ Fayet-Iliopoulos term as the Lagrangian densities (like in other PBGS cases, these two independent invariants are reduced to each other after passing to the nonlinear realization).

4 Space-filling D3-brane

As the last example we consider the space-filling D3-brane in $d=4$. This system amounts to the PBGS pattern $N=2 \rightarrow N=1$ in $d=4$, with a nonlinear generalization of $N=1$, $d=4$ vector multiplet as the Goldstone multiplet [5, 6]. The off-shell superfield action for this system was constructed in [21, 5]. Here we explain, following ref. [11], how the corresponding dynamical equations can be derived directly from the coset approach, like in other cases considered in this paper. As a new reuslt, we shall recover the action of [5] by means of the universal procedure exemplified in the previous Sections, thus establishing a direct link between the approach of refs. [5, 6] and the customary coset approach.

4.1 D3-brane superfield equations of motion from nonlinear realizations. Our starting point is the $N=2$, $d=4$ Poincaré superalgebra without central charges:

$$\left\{Q_\alpha, \bar{Q}_{\dot\alpha}\right\} = 2P_{\alpha\dot\alpha}\ , \ \left\{S_\alpha, \bar{S}_{\dot\alpha}\right\} = 2P_{\alpha\dot\alpha}\ . \tag{4.14}$$

Assuming the $S_\alpha, \bar{S}_{\dot\alpha}$ supersymmetries to be spontaneously broken, we introduce the Goldstone superfields $\psi^\alpha(x,\theta,\bar\theta)$, $\bar\psi^{\dot\alpha}(x,\theta,\bar\theta)$ as the corresponding parameters in the following coset

$$g = e^{ix^{\alpha\dot\alpha}P_{\alpha\dot\alpha}} e^{i\theta^\alpha Q_\alpha + i\bar\theta_{\dot\alpha}\bar{Q}^{\dot\alpha}} e^{i\psi^\alpha S_\alpha + i\bar\psi_{\dot\alpha}\bar{S}^{\dot\alpha}}\ . \tag{4.15}$$

With the help of the corresponding Cartan forms one can define the covariant derivatives

$$\mathcal{D}_\alpha = D_\alpha - i\left(\bar\psi^{\dot\beta}D_\alpha\psi^\beta + \psi^\beta D_\alpha\bar\psi^{\dot\beta}\right)\mathcal{D}_{\beta\dot\beta}, \quad \mathcal{D}_{\alpha\dot\alpha} = \left(E^{-1}\right)^{\beta\dot\beta}_{\alpha\dot\alpha}\partial_{\beta\dot\beta}\ , \tag{4.16}$$

where

$$D_\alpha = \frac{\partial}{\partial\theta^\alpha} - i\bar\theta^{\dot\alpha}\partial_{\alpha\dot\alpha}\ , \ \ \bar{D}_{\dot\alpha} = -\frac{\partial}{\partial\bar\theta^{\dot\alpha}} + i\theta^\alpha\partial_{\alpha\dot\alpha}\ , \ \ E^{\beta\dot\beta}_{\alpha\dot\alpha} = \delta^\beta_\alpha\delta^{\dot\beta}_{\dot\alpha} - i\psi^\beta\partial_{\alpha\dot\alpha}\bar\psi^{\dot\beta} - i\bar\psi^{\dot\beta}\partial_{\alpha\dot\alpha}\psi^\beta\ . \tag{4.17}$$

Now we can write the covariant version of the constraints on ψ^α, $\bar\psi^{\dot\alpha}$ which define the superbrane generalization of $N=1$, $d=4$ vector multiplet, together with the covariant equations of motion for this system. They are a direct covariantization of the free $N=1$, $d=4$ Maxwell superfield strength constraints and equation of motion:

$$(a)\ \ \overline{\mathcal{D}}_{\dot\alpha}\psi_\alpha = 0\ , \ \ \mathcal{D}_\alpha\bar\psi_{\dot\alpha} = 0\ , \qquad (b)\ \ \mathcal{D}^\alpha\psi_\alpha = 0\ , \ \ \overline{\mathcal{D}}_{\dot\alpha}\overline{\psi}^{\dot\alpha} = 0\ . \tag{4.18}$$

Eqs. (4.18a) are a covariantization of the flat $N=1$ chirality conditions while (4.18b) generalizes at once the $N=1$ superfield strength Bianchi identity and equation of motion. As was argued in [11], this set of superfield equations is self-consistent and compatible with the algebra of the covariant derivatives (4.16). For the physical bosonic components of $\psi, \bar\psi$,

$$V^{\alpha\beta} \equiv \mathcal{D}^\alpha\psi^\beta|_{\theta=0}\ , \quad \bar{V}^{\dot\alpha\dot\beta} \equiv \overline{\mathcal{D}}^{\dot\alpha}\bar\psi^{\dot\beta}|_{\theta=0}, \tag{4.19}$$

these superfield equations imply, in the purely bosonic limit, the following equations

$$\partial_{\alpha\dot\alpha}V^{\alpha\beta} - V^\gamma_\alpha\bar{V}^{\dot\gamma}_{\dot\alpha}\,\partial_{\gamma\dot\gamma}V^{\alpha\beta} = 0\ , \quad \partial_{\alpha\dot\alpha}\bar{V}^{\dot\alpha\dot\beta} - V^\gamma_\alpha\bar{V}^{\dot\gamma}_{\dot\alpha}\,\partial_{\gamma\dot\gamma}\bar{V}^{\dot\alpha\dot\beta} = 0\ . \tag{4.20}$$

It was shown in [11] that, like the analogous equations (3.4) in the D2-brane case, these equations can be cast in the standard form of the $d=4$ BI theory equations augmented with the Bianchi identity for the Maxwell field strength.

Note that at the full superfield level the field redefinition which leads from the disguised form of the BI equations (4.20) to their "canoical" form corresponds to passing from the Goldstone fermions ψ_α, $\bar\psi_{\dot\alpha}$ to the standard Maxwell superfield strength $W_\alpha, \bar W_{\dot\alpha}$. The nonlinear action of [21, 5, 6] was written just in terms of this latter object. The equivalent form (4.18) of the equations of motion and Bianchi identity is advantageous in that it manifests the second (hidden) supersymmetry, being constructed out of the covariant objects.

4.2 Linear and nonlinear realizations of the $N = 2 \rightarrow N = 1$ PBGS. Now we wish to precisely establish the correspondence just mentioned and to reproduce the off-shell BI action of [21, 5, 6] by applying the general techniques based on the relationship between linear and nonlinear realizations of PBGS, like in the previous Sections.

Our starting point is the $N = 2, d = 4$ Goldstone-Maxwell multiplet [14, 5, 15]. In the $N = 2$ superspace $(x^{\alpha\dot\alpha}, \theta^\alpha_i, \bar\theta^{\dot\alpha i})$ it is defined by the following deformation [15] of the standard $N = 2$ Maxwell superfield strength constraints

$$\text{(a)}\ \ D^{ik}W - \bar D^{ik}\bar W = iM^{(ik)}\ , \qquad \text{(b)}\ \ D^i_\alpha \bar W = \bar D_{\dot\alpha i} W = 0\ . \tag{4.21}$$

Here

$$D^i_\alpha = \frac{\partial}{\partial\theta^\alpha_i} - i\bar\theta^{\dot\alpha i}\partial_{\alpha\dot\alpha}\ ,\ \bar D_{\dot\alpha i} = -\frac{\partial}{\partial\bar\theta^{\dot\alpha i}} + i\theta^\alpha_i\partial_{\alpha\dot\alpha}\ ,\ D^{ik} = D^{\alpha i}D^k_\alpha\ ,\ \bar D^{ik} = \bar D^i_{\dot\alpha}\bar D^{\dot\alpha k}$$

and $M^{ik} = M^{ki}$ is a triplet of constants which explicitly break the automorphism $SU(2)_A$ of $N = 2$ supersymmetry down to $U(1)_A$ and satisfy the pseudo-reality condition

$$\overline{(M^{ik})} = \epsilon_{in}\epsilon_{km}M^{nm}\ .$$

In components, the deformation (4.21a) amounts to the appearance of constant imaginary part $\sim M^{ik}$ in the isotriplet auxiliary field of $N = 2$ Maxwell multiplet.

Now we pass to the $N = 1$ superfield notation by relabelling the Grassmann coordinates and spinor derivatives as

$$\theta^\alpha_1 \equiv \theta^\alpha\ ,\ \theta^\alpha_2 \equiv \zeta^\alpha,\ D^1_\alpha \equiv D_\alpha,\ D^2_\alpha \equiv D^\zeta_\alpha\ .$$

In order to have the off-shell S-supersymmetry (acting as ζ-supertranslations) spontaneously broken while the Q-supersymmetry unbroken, we are led to choose the following frame with respect to the explicitly broken $SU(2)_A$

$$M^{12} = 0\ , \quad M^{11} = M^{22} = m\ , \tag{4.22}$$

where m is a real constant. Like in the case of D2-brane it is fixed up to rescaling of W. A convenient choice is

$$m = -2\ .$$

It will be also convenient to choose the basis in $N = 2$ superspace where the chirality with respect to the variable ζ^α is manifest

$$\bar D^\zeta_{\dot\alpha} = -\frac{\partial}{\partial\bar\zeta^{\dot\alpha}}\ , \quad D^\zeta_\alpha = \frac{\partial}{\partial\zeta^\alpha} - 2i\bar\zeta^{\dot\alpha}\partial_{\alpha\dot\alpha}\ . \tag{4.23}$$

In this basis, constraints (4.21) imply the following structure of the superfield $W(x,\theta,\zeta)$

$$W = i\phi + i\zeta^\alpha W_\alpha - i\frac{1}{2}\zeta^2\left(1+\frac{1}{2}\bar{D}^2\bar{\phi}\right) \ , \tag{4.24}$$

where ϕ and W_α are chiral $N=1$ superfields

$$\bar{D}_{\dot\alpha}\phi = \bar{D}_{\dot\alpha}W_\alpha = 0 \ , \tag{4.25}$$

and the fermionic superfield W_α obeys the $N=1$ Maxwell superfield strength constraint

$$D^\alpha W_\alpha + \bar{D}_{\dot\alpha}\bar{W}^{\dot\alpha} = 0 \ . \tag{4.26}$$

The numerical factors in (4.24) were chosen for further convenience.

The S-supersymmetry transformation of the $N=2$ superfield W

$$\delta_\eta W = -\left[\eta^\alpha\frac{\partial}{\partial\zeta^\alpha} + \bar\eta^{\dot\alpha}\left(\frac{\partial}{\partial\bar\zeta^{\dot\alpha}} + 2i\zeta^\alpha\partial_{\alpha\dot\alpha}\right)\right]W \tag{4.27}$$

implies the following ones for its $N=1$ superfield components ϕ and W_α

$$\begin{aligned}
&\delta_\eta\phi = -(\eta W) \ , \quad \delta_\eta\bar\phi = -(\bar W\bar\eta) \ , \\
&\delta_\eta W_\alpha = \eta_\alpha\left(1+\frac{1}{2}\bar{D}^2\bar\phi\right) + 2i\bar\eta^{\dot\alpha}\partial_{\alpha\dot\alpha}\phi \ , \quad \delta_\eta\bar W_{\dot\alpha} = \overline{(\delta_\eta W_\alpha)} \ .
\end{aligned} \tag{4.28}$$

The superfield W_α shows up an inhomogeneous shift $\sim\eta_\alpha$ (proportional to the $SU(2)_A$ breaking parameters) in its transformation, so it is the Goldstone fermion of the linear realization of the considered $N=2\to N=1,\ d=4$ PBGS pattern (the Goldstone-Maxwell $N=1$ superfield).

Now we are prepared to start the algorithmic procedure of passing to the relevant nonlinear realization exemplified in the previous Sections. We construct the finite η-transformations of the superfields ϕ and W_α proceeding from the infinitesimal ones (4.28)

$$\{\,\phi(\eta)\ ,\ \ W_\alpha(\eta)\,\} = \left(1+\delta_\eta+\frac{1}{2}\delta_\eta^2+\frac{1}{3!}\delta_\eta^3+\frac{1}{4!}\delta_\eta^4\right)\{\,\phi\ ,\ \ W_\alpha\,\} \ , \tag{4.29}$$

then pull out the parameters $\eta_\alpha,\bar\eta_{\dot\alpha}$ to the left and replace them by the original nonlinear realization Goldstone fermions, $\eta_\alpha\to-\psi_\alpha,\ \bar\eta_{\dot\alpha}\to-\bar\psi_{\dot\alpha}$. It is a matter of straightforward computation to check that the objects $\tilde\phi\equiv\phi(-\psi),\tilde W_\alpha\equiv W_\alpha(-\psi)$ transform homogeneously (though nonlinearly) with respect to the η-transformations

$$\delta_\eta\{\,\tilde\phi,\ \tilde W_\alpha\,\} = i\left(\psi^\alpha\bar\eta^{\dot\alpha}-\eta^\alpha\bar\psi^{\dot\alpha}\right)\partial_{\alpha\dot\alpha}\{\,\tilde\phi,\ \tilde W_\alpha\,\} \ , \tag{4.30}$$

and behave as ordinary $N=1$ superfields under the unbroken ϵ-supertranslations acting in the $N=1$ superspace $(x,\theta,\bar\theta)$. Hence, one can impose the covariant constraints

$$\tilde\phi = \tilde W_\alpha = 0 \ . \tag{4.31}$$

Explicitly, the relations between ϕ, W_α and ψ_α implied by these constraints are as follows

$$\phi = -\frac{1}{2}\psi^2\left(1+\frac{1}{2}\bar D^2\bar\phi\right) - i\psi^\alpha\bar\psi^{\dot\alpha}\partial_{\alpha\dot\alpha}\phi - i\psi^2\bar\psi_{\dot\alpha}\partial^{\dot\alpha\beta}W_\beta - \frac{3}{8}\psi^4\Box\phi \ , \tag{4.32}$$

$$\begin{aligned}
W_\alpha &= \psi_\alpha\left(1+\frac{1}{2}\bar D^2\bar\phi\right) + 2i\bar\psi^{\dot\alpha}\partial_{\alpha\dot\alpha}\phi + i\left(\psi_\alpha\bar\psi^{\dot\alpha}\partial_{\beta\dot\alpha}W^\beta + \psi_\beta\bar\psi^{\dot\alpha}\partial_{\alpha\dot\alpha}W^\beta\right) \\
&+\frac{1}{2}\left(\bar\psi^2\psi_\alpha\Box\phi - \frac{i}{2}\psi^2\bar\psi^{\dot\alpha}\partial_{\alpha\dot\alpha}\bar D^2\bar\phi\right) - \frac{1}{8}\psi^4\Box W_\alpha \ , \quad \Box\equiv\partial_{\alpha\dot\alpha}\partial^{\dot\alpha\alpha} \ .
\end{aligned} \tag{4.33}$$

These equations look a bit more complicated as compared to the previous examples, but, nevertheless, they can be treated in the precisely same algorithmic way. One should firstly make use of the relation (4.33) (and its conjugate) to express ψ_α, $\bar{\psi}$ in terms of W_α, $\bar{W}_{\dot\alpha}$ and their x-derivatives, and then substitute these expressions into (4.32) and its conjugate, thus obtaining covariant relations between $\phi, \bar\phi$ and $W_\alpha, \bar{W}_{\dot\alpha}$. The latter should allow one to trade $\phi, \bar\phi$ for $W_\alpha, \bar{W}_{\dot\alpha}$ (or for $\psi_\alpha, \bar\psi_{\dot\alpha}$ in view of the equivalence relation between these two kinds of the Goldstone fermion). A technically more simple way to arrive at the final relations is as follows. One computes W^2 from (4.33) and in the obtained relation

$$W^2 = \psi^2 \left(1 + \frac{1}{2}\bar{D}^2\bar\phi\right)^2 + \ldots$$

eliminates all x-derivatives of ϕ and W_α (denoted by $\ldots$) in terms of those of ψ_α using the nilpotency properties of ψ_α, $\bar\psi_{\dot\alpha}$ and the reduction formulas like

$$\bar\psi^2(\partial\phi\cdot\partial\phi) = -\frac{1}{2}\psi^4(\partial\psi^\alpha\cdot\partial\psi_\alpha)\left(1+\frac{1}{2}\bar{D}^2\bar\phi\right)^2 \;, \quad \psi^4\Box\phi = -\psi^4\,(\partial\psi^\alpha\cdot\partial\psi_\alpha)\left(1+\frac{1}{2}\bar{D}^2\bar\phi\right) \;,$$

which also follow from (4.32), (4.33). The same procedure is applied to similar terms in (4.32). As the next step, one substitutes $\psi^2 = W^2\left(1+\frac{1}{2}\bar{D}^2\bar\phi\right)^{-2} + \ldots$ into (4.32). All "superfluous" terms are canceled among themselves and one ends up with the simple relations

$$\phi = -\frac{1}{2}\,\frac{W^2}{1+\frac{1}{2}\bar{D}^2\bar\phi} \;, \quad \bar\phi = -\frac{1}{2}\,\frac{\bar{W}^2}{1+\frac{1}{2}D^2\phi} \;, \tag{4.34}$$

which are just those postulated in [5] and derived from the nilpotency condition in [6]. We see that the same relations follow from our generic procedure applied to the given specific case. An advantage of this derivation is that it sets the direct relationship with the "canonical" nonlinear realization through the equations (4.32), (4.33). In particular, notice the relation

$$\psi^4 = \psi^2\bar\psi^2 = W^2\bar{W}^2\left[\left(1+\frac{1}{2}\bar{D}^2\bar\phi\right)\left(1+\frac{1}{2}D^2\phi\right)\right]^{-2} \;.$$

As was shown in [5, 6] the chiral superfield ϕ is just the Goldstone superfield Lagrangian density for the $N=2 \to N=1$ PBGS (it is the Fayet-Iliopoulos term from the $N=2$ perspective). It describes a $N=1$ superextension of the $d=4$ BI theory with the second hidden $N=1$ supersymmetry, or, equivalently, the gauge-fixed space-filling D3-brane in a flat background. For completeness, we quote here the solution of (4.34) [5]

$$\phi = -\frac{1}{2}\left\{W^2 + \frac{1}{2}\bar{D}^2\,\frac{W^2\bar{W}^2}{1-\frac{1}{2}A+\sqrt{1-A+\frac{1}{4}B^2}}\right\} \tag{4.35}$$

$$A \equiv \frac{1}{2}\left(D^2W^2 + \bar{D}^2\bar{W}^2\right) \;, \quad B \equiv \frac{1}{2}\left(D^2W^2 - \bar{D}^2\bar{W}^2\right) \;. \tag{4.36}$$

Having at our disposal the explicit relations (4.32), (4.33) we can in principle explicitly check, along the lines of Subsect. 2.2, the equivalence between the equations of motion corresponding to the $N=2 \to N=1$ BI Lagrangian (4.35) and eqs. (4.18) proposed within the original nonlinear realization setting.

5 Concluding remarks

In this contribution we reviewed basic features of the PBGS approach to superbranes and presented a few novel developments. In particular, we showed that the method of constructing Goldstone superfield actions which is based on the general relationship between linear and nonlinear realizations of PBGS [26, 17, 18] is fairly workable not only in the simple examples treated in this way earlier [17, 18], but also in some more complicated and interesting cases including the space-filling D3-brane (the $N = 2 \to N = 1$ BI theory). In this short review we left aside such interesting cases as the $N = 4 \to N = 2$ and $N = 8 \to N = 4$ BI theories (super D3- and D6-branes in $D = 6$ and $D = 10$) [12, 13] which certainly offer new domains for applying the machinery expounded here. A further work is also required in order to understand in full the links between the PBGS and superembedding [29] approaches. In recent papers [30, 32, 31], the $N = 1$, $D = 4$ supermembrane and D2-brane PBGS actions (2.19), (2.20) and (3.12), (3.13) originally derived in [7] were recovered from the superembedding approach, and some steps toward a similar derivation of the PBGS D3-brane action of ref.[5] were undertaken. It would be tempting to understand linear realizations of the PBGS theories and their relationship to nonlinear realizations from the superembedding point of view.

Acknowledgments

I thank S. Bellucci and S. Krivonos in collaboration with whom some of the results mentioned here were obtained. Useful discussions with them, as well as with F. Delduc, A. Kapustnikov, O. Lechtenfeld, D. Sorokin, K. Stelle and B. Zupnik are kindly acknowledged. I am grateful to the Organizers of the Misha Saveliev Seminar in Protvino for inviting me to give a talk and for kind hospitality. This work was supported in part by the RFBR-CNRS Grant No. 98-02-22034, RFBR Grant No. 99-02-18417 and NATO Grant No. PST.CLG 974874.

References

[1] J. Bagger, J. Wess, Phys. Lett. **B 138** (1984) 105.

[2] J. Hughes, J. Liu, J. Polchinski, Phys. Lett. **B 180** (1986) 370; J. Hughes, J. Polchinski, Nucl. Phys. **B 278** (1986) 147.

[3] A.Achucarro, J. Gauntlett, K. Itoh, P.K. Townsend, Nucl. Phys. **B 314** (1989) 129.

[4] J. Bagger, A. Galperin, Phys. Lett. **B 336** (1994) 25; *ibid*, **B 412** (1997) 296.

[5] J. Bagger, A. Galperin, Phys. Rev. **D 55** (1997) 1091.

[6] M. Roček, A. Tseytlin, Phys. Rev. **D 59** (1999) 106001.

[7] E. Ivanov, S. Krivonos, Phys. Lett. **B 453** (1999) 237.

[8] S. Bellucci, E. Ivanov, S. Krivonos, Phys. Lett. **B 460** (1999) 348.

[9] S. Bellucci, E. Ivanov, S. Krivonos, In: Proceedings of 32nd International Symposium Ahrenshoop, September 1 - 5, 1998, Buckow, Germany, Fortsch. Phys. **48** (2000) 19.

[10] P. West, *" Automorphisms, Non-linear Realizations and Branes"*, `hep-th/0001216`.

[11] S. Bellucci, E. Ivanov, S. Krivonos, Phys. Lett. **B 482** (2000) 233.

[12] S. Bellucci, E. Ivanov, S. Krivonos, Phys. Lett. **B 502** (2001) 279.

[13] S. Bellucci, E. Ivanov, S. Krivonos, *" Towards the complete N=2 superfield Born-Infeld action with partially broken N=4 supersymmetry"*, `hep-th/0101195`.

[14] I. Antoniadis, H. Partouche, T.R. Taylor, Phys. Lett. **B 372** (1996) 83.

[15] E.A. Ivanov, B.M. Zupnik, Yad. Fiz. **62** (1999) 1110.

[16] B.M. Zupnik, Phys. Lett. **B 461** (1999) 203; Teor. Mat. Fiz. **123** (2000) 57.

[17] F. Delduc, E. Ivanov, S. Krivonos, Nucl. Phys. **B 576** (2000) 196.

[18] E. Ivanov, S. Krivonos, O. Lechtenfeld, B. Zupnik, Nucl. Phys. **B 600** (2001) 235.

[19] A. Kapustnikov, A. Shcherbakov, *"Linear and Nonlinear Realizations of Superbranes"*, `hep-th/0104196`.

[20] A.A. Tseytlin, *"Born-Infeld Action, Supersymmetry and String Theory"*, `hep-th/9908105`.

[21] S. Cecotti, S. Ferrara, Phys. Lett. **B 187** (1987) 335.

[22] S. Ketov, Mod. Phys. Lett. **A 14** (1999) 501; Class. Quant. Grav. **17** (2000) L91.

[23] S.M. Kuzenko, S. Theisen, Fortsch. Phys. **49** (2001) 273; JHEP **0003** (2000) 034.

[24] M. Aganagic, C. Popescu, J.H. Schwarz, Nucl. Phys. **B 495** (1997) 99.

[25] S. Coleman, J. Wess, B. Zumino, Phys. Rev. **177** (1999) 2239; C. Callan, S. Coleman, J. Wess, B. Zumino, *ibid*, **177** (1969) 2247; D.V. Volkov, Sov. J. Part. Nucl. **4** (1973) 3; V.I. Ogievetsky, In: *Proceedings of Xth Winter School of Theoretical Physics in Karpacz*, Acta Universitatis Wratislaviensis, Vol. **207**, Wroclaw, 1974, p.227.

[26] E. Ivanov, A. Kapustnikov, J. Phys. **A 11** (1978) 2375; Nucl. Phys. **B 333** (1990) 439.

[27] E.A. Ivanov, V.I. Ogievetsky, Teor. Mat. Fiz. **25** (1975) 164.

[28] E.A. Ivanov, S.O. Krivonos, V.M. Leviant, J. Phys. **A 22** (1989) 4201.

[29] D. Sorokin, Phys. Reports **329** (2000) 1.

[30] P. Pasti, D. Sorokin, M. Tonin, Nucl. Phys. **B 591** (2000) 109; *"Geometrical aspects of superbrane dynamics"*, `hep-th/0011020`.

[31] I. Bandos, P. Pasti, A. Pokotilov, D. Sorokin, M. Tonin, *"Space Filling Dirichlet 3-Brane in* $N=2, D=4$ *Superspace"*, `hep-th/0103152`.

[32] J.M. Drummond, P.S. Howe, *"Codimension zero superembeddings"*, `hep-th/0103191`.